Late-time evolution of the Yang-Mills field in the spherically symmetric gravitational collapse

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Abstract

We investigate the late-time evolution of the Yang-Mills field in the self-gravitating backgrounds: Schwarzschild and Reissner-Nordström spacetimes. The late-time power-law tails develop in the three asymptotic regions: the future timelike infinity, the future null infinity and the black hole horizon. In these two backgrounds, however, the late-time evolution has quantitative and qualitative differences. In the Schwarzschild black hole background, the late-time tails of the Yang-Mills field are the same as those of the neutral massless scalar field with multipole moment $l = 1$. The late-time evolution is dominated by the spacetime curvature. When the background is the Reissner-Nordström black hole, the late-time tails have not only a smaller power-law exponent, but also an oscillatory factor. The late-time evolution is dominated by the self-interacting term of the Yang-Mills field. The cause responsible for the differences is revealed.

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I. INTRODUCTION

In the process of gravitational collapse, due to the backscattering off the spacetime curvature, the perturbations outside a star or a black hole will die off in the form of a inverse power-law tail. As a feature of the late-time evolution of gravitational collapse, the power-law tail has been studied by many authors.

The late-time evolution of a massless scalar field on a fixed Schwarzschild background was investigated first by Price [1]. He found that an initially static l pole dies off as $t^{-(2l+2)}$, while it must fall off as $t^{-(2l+3)}$ if there is no initial l pole but one develops during the collapse. Here t is the Schwarzschild coordinate time. The linearized electromagnetic and gravitational perturbations also satisfy the Klein-Gordon equation with a somewhat different effective potential. So the electromagnetic and gravitational perturbations have the similar late-time behavior as that of the scalar field. The late-time power-law tail develops not only at timelike infinity, but also at null infinity and along the event horizon of black holes [2]. Furthermore, the power-law tail occurs even when no horizon is present in the background. This implies that the power-law tail should be present in perturbations of stars, or after the implosion and subsequent explosion of a massless field which does not result in the formation of black hole. Indeed this has been confirmed numerically in [3]. The late-time behavior can also be approached by employing the spectral decomposition of corresponding Green's functions [3–5].

Recently, Hod and Piran have studied the late-time behavior of a massless charged scalar field [6,7], and a massive scalar field [8] in the gravitational collapse. Some differences of significance have been observed between the massless neutral scalar field and charged scalar field. In particular, they found that the late-time tail of charged scalar field has an extra oscillatory factor along the black hole horizon. Due to the interaction of electromagnetic field, the power-law exponents of the late-time tails are smaller than those of neutral scalar field. Therefore, they concluded that a charged black hole becomes bald slower than a neutral one. It is of importance to note that, contrary to the neutral scalar field, whose late-time evolution is dominated by the spacetime curvature, the late-time evolution of charged scalar field is dominated by the electromagnetic interaction, an effect in a flat spacetime.

According to the no hair theorem of black holes, the collapse of a missive body may lead to the formation of a black hole and the external gravitational field of the black hole settles down to the Kerr-Newmann family, which characterized by only three parameters: mass, charge and angular momentum. Indeed, it has been proved that there do not exist nontrivial neutral or charged scalar field outside black holes [9]. In this sense, the late-time power-law tail of scalar fields in fact shows a dynamical mechanism by which the scalar fields are radiated away in the gravitational collapse, or perturbations die off. In addition, the form of the late-time tail is closely revelant to the internal structure of black holes. The late-time tail will act as an input in the study of evolution inside black holes. For instance, the late-time tail must be used in the mass inflation scenario [10] and in the study of Cauchy horizon stability of black holes.

In the present paper we would like to study the late-time behavior of the Yang-Mills field in the self-gravitating background. Since the discovery of the particle-like solution by Bartnik and McKinnon [11], the Einstein-Yang-Mills system (and its generalizations) has drawn a great deal of interest. In particular, the so-called colored black hole has been found

[12], which violates the no-hair theorem of black holes. That is, the Yang-Mills (YM) field can be regarded as a kind of hairs of black holes. In addition, the Yang-Mills field has a self-interacting term. We expect that it may give rise to some interesting phenomena.

The plan of this paper is as follows. For completeness, in the next section we briefly introduce the method of spectral decomposition of Green's function. It already proves that the Green's function technique is a powerful tool to study the dynamical evolution of fields. In Sec. III we linearize the equations of motion for the Einstein-Yang-Mills system and obtain the linearized equation of the Yang-Mills field. In Sec. IV and V we study the late-time behavior of the Yang-Mills field in the Schwarzschild and Reissner-Nordström black hole backgrounds, respectively. Our main results are summarized in Sec. VI.

II. SPECTRAL DECOMPOSITION OF EVOLUTION FIELDS

Consider a perturbation field denoted by Φ, which satisfies the following equation

$$[\partial_t^2 - \partial_y^2 + V(y)]\Phi(y,t) = 0. \tag{2.1}$$

In order to analytically study the dynamical evolution of the field Φ in the potential $V(y)$, it is convenient to use the Green's function techniques. The evolution of Φ can be determined by the Green's function and initial conditions as

$$\Phi(y,t) = \int [G(y,x;t)\partial_t\Phi(x,0) + \partial_t G(y,x;t)\Phi(x,0)]dx, \tag{2.2}$$

for $t \geq 0$. The retarded Green's function $G(y,x;t)$ obeys the equation

$$[\partial_t^2 - \partial_y^2 + V(y)]G(y,x;t) = \delta(t)\delta(y-x). \tag{2.3}$$

subject to the condition $G(y,x;t) = 0$ for $t \leq 0$. In order to get the Green's function, one may use the Fourier transform

$$\tilde{G}(y,x;\sigma) = \int_{0^-}^{\infty} G(y,x;t)e^{i\sigma t}dt. \tag{2.4}$$

This Fourier transform is well-defined in the upper half σ plane and the Green's function $\tilde{G}(y,x;\sigma)$ satisfies

$$[\partial_y^2 + \sigma^2 - V(y)]\tilde{G}(y,x;\sigma) = \delta(y-x). \tag{2.5}$$

Thus, once given the Green's function $\tilde{G}(y,x;\sigma)$, one can obtain the Green's function $G(y,x;t)$ using the inversion transform

$$G(y,x;t) = \frac{1}{2\pi}\int_{-\infty+ic}^{\infty+ic} \tilde{G}(y,x;\sigma)e^{-i\sigma t}d\sigma, \tag{2.6}$$

where c is some positive constant. To construct $\tilde{G}(y,x;\sigma)$, one may use two functions ϕ_i, which are two linearly independent solutions to the homogeneous equation

$$[\partial_y^2 + \sigma^2 - V(y)]\phi_i(y,\sigma) = 0, \quad i = 1, 2. \tag{2.7}$$

These two functions ϕ_i must satisfy appropriate boundary conditions. For asymptotically flat black hole spacetimes, they should have following asymptotic behaviors

$$\phi_1(y,\sigma) \sim \begin{cases} e^{-i\sigma y}, & y \to -\infty, \\ A_{\rm out}(\sigma)e^{i\sigma y} + A_{\rm in}(\sigma)e^{-i\sigma y}, & y \to +\infty. \end{cases} \tag{2.8}$$

and

$$\phi_2(y,\sigma) \sim \begin{cases} B_{\rm out}(\sigma)e^{i\sigma y} + B_{\rm in}(\sigma)e^{-i\sigma y}, & y \to -\infty, \\ e^{i\sigma y}, & y \to +\infty. \end{cases} \tag{2.9}$$

That is, ϕ_1 has only a purely ingoing wave crossing the black hole horizon ($y \to -\infty$). According to the coefficients in (2.8), the transmission and reflection amplitudes are

$$\mathcal{T}_1(\sigma) = \frac{1}{A_{\rm in}(\sigma)}, \quad \mathcal{R}_1(\sigma) = \frac{A_{\rm out}(\sigma)}{A_{\rm in}(\sigma)}. \tag{2.10}$$

ϕ_2 has only a purely outgoing wave at spatial infinity. The transmission and reflection amplitudes are

$$\mathcal{T}_2(\sigma) = \frac{1}{B_{\rm out}(\sigma)}, \quad \mathcal{R}_2(\sigma) = \frac{B_{\rm in}(\sigma)}{B_{\rm out}(\sigma)}. \tag{2.11}$$

Therefore, while (2.10) gives the absorption coefficient, $T(\sigma) = |\mathcal{T}_1(\sigma)|^2$, of the black hole, (2.11) gives the Hawking radiation coefficient of the black hole, $R(\sigma) = |\mathcal{R}_2(\sigma)|^2$.

Using the two functions, the Green's function $\tilde{G}(y,x;\sigma)$ can be expressed as

$$\tilde{G}(y,x;\sigma) = -\frac{1}{W(\sigma)} \begin{cases} \phi_1(y,\sigma)\phi_2(x,\sigma), & y < x, \\ \phi_1(x,\sigma)\phi_2(y,\sigma), & y > x, \end{cases} \tag{2.12}$$

where $W(\sigma)$ is the Wronskian of ϕ_i, defined as

$$W(\sigma) = \phi_1(y,\sigma)\partial_y\phi_2(y,\sigma) - \phi_2(y,\sigma)\partial_y\phi_1(y,\sigma) = 2i\sigma A_{\rm in}(\sigma). \tag{2.13}$$

The Wronskian is independent of y.

To get the Green's function $G(y,x;t)$ in (2.6), we must choose an appropriate integration contour. Usually one may bend the integration contour into the lower half of the complex σ plane. In this way, one can isolate the behavior of the Green's function in the different time intervals. The Green's function consists of three parts [4,5,7].

(1) *Prompt response.* This part comes from the integral along the large semi-circle. so it corresponds to the high-frequency response. In the high-frequency limit the Green's function becomes the propagator in flat spacetime. This means that the radiation reaches the observer directly from the source. This is therefore a short-time response and will die off beyond some time.

(2) *Quasinormal modes.* The Green's function $\tilde{G}(y,x;\sigma)$ has an infinite number of distinct singularities in the lower half plan of the complex σ. These singularities correspond to the black hole quasinormal modes and they occur when the Wronskian vanishes there. This part falls off exponentially because of Im$\sigma < 0$ for each mode.

(3) *Late-time tail.* Following the quasinormal modes is just the late-time tail. This part is associated with the existence of a branch cut in the solution ϕ_2 in this complex picture. This cut is usually placed along the negative imaginary σ axis. The contribution of this part to the Green's function comes from the integral around the branch cut. As was shown previously, this part generally has an inverse power-law form in the asymptotically flat spacetimes [1].

In this paper we just study the late-time tail of the YM field in its own gravitational background. So in the next section we first obtain the linearized YM equation.

III. LINEARIZED EINSTEIN-YANG-MILLS EQUATIONS

Consider the gravitational collapse of the Yang-Mills field, whose dynamics is governed by the action

$$S = \frac{1}{16\pi} \int d^4x\sqrt{-g}[R - F_{\mu\nu}F^{\mu\nu}], \tag{3.1}$$

where R denotes the curvature scalar and $F_{\mu\nu}$ is the Yang-Mills field strength defined as $F = dA + A \wedge A$. Here A is the Yang-Mills potential. Due to the conformal invariance, the Einstein equations can be written down as

$$R_{\mu\nu} = 2F_{\mu\lambda}F_{\nu}{}^{\lambda} - \frac{1}{2}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}, \tag{3.2}$$

and the equation of Yang-Mills field is $D^*F = 0$. We now consider the spherically symmetric gravitational collapse. So the line element can be written as

$$ds^2 = -e^{\nu}dt^2 + e^{\lambda}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2. \tag{3.3}$$

Correspondingly, we take the following ansatz for the Yang-Mills potential:

$$A = w\tau_1 d\theta + (\cot\theta\tau_3 + w\tau_2)\sin\theta d\phi. \tag{3.4}$$

Here τ_i ($i = 1, 2, 3$) are standard generators of su(2) Lie algebra. ν, λ and w are functions of r and t, and, for simplicity, we have already set the electric components of the Yang-Mills potential vanish. In the metric (3.3), the Einstein equations can be simplified to

$$\lambda' + \nu' = \frac{4}{r}(w'^2 + \dot{w}^2 e^{-\nu+\lambda}), \tag{3.5}$$

$$\dot{\lambda} = \frac{4w'\dot{w}}{r}, \tag{3.6}$$

$$1 - e^{-\lambda} + \frac{re^{-\lambda}}{2}(\lambda' - \nu') = \frac{(1-w^2)^2}{r^2}, \tag{3.7}$$

and the equation of the Yang-Mills potential w is

$$\ddot{w}e^{\lambda-\nu} + \frac{\dot{\lambda} - \dot{v}}{2}\dot{w}e^{\lambda-\nu} - w'' - \frac{\nu' - \lambda'}{2}w' - \frac{(1-w^2)w}{r^2}e^{\lambda} = 0, \tag{3.8}$$

where a prime represents derivative with respect to r and an overdot stands for derivative with respect to t. Here we mention that the critical behavior of the gravitational collapse of the YM field has been studied in [13], there two distinct critical solutions have been found numerically.

To study the late-time behavior of the Yang-Mills field in the process of the gravitational collapse, we now linearize the Einstein-Yang-Mills equations. Suppose that the final static background is described by functions ν_0, λ_0 and w_0, which depend on r only. The functions ν, λ, and w can be expanded as

$$\nu = \nu_0 + \nu_1, \quad \lambda = \lambda_0 + \lambda_1, \quad w = w_0 + w_1. \tag{3.9}$$

Thus we obtain the linearized equations

$$\lambda_1' + \nu_1' = \frac{8w_0'}{r} w_1', \tag{3.10}$$

$$\dot{\lambda}_1 = \frac{4w_0'}{r} \dot{w}_1, \tag{3.11}$$

$$\lambda_1' - \nu_1' + (\frac{2}{r} - \lambda_0' + \nu_0')\lambda_1 + \frac{8e^{\lambda_0}}{r^3}(w_0 - w_0^3)w_1 = 0, \tag{3.12}$$

and

$$w_1'' + \frac{\nu_0' - \lambda_0'}{2} w_1' + \frac{w_0'}{2}(\nu_1' - \lambda_1') + \frac{(w_0 - w_0^3)}{r^2} e^{\lambda_0}\lambda_1 + \frac{1 - 3w_0^2}{r^2} e^{\lambda_0} w_1 - e^{\lambda_0 - \nu_0}\ddot{w}_1 = 0. \tag{3.13}$$

Using (3.10)-(3.12), and defining $w_1 = e^{-i\sigma t} e^{(\lambda_0 - \nu_0)/4}\phi(r)$, we have

$$\begin{aligned}\Bigg[\frac{d^2}{dr^2} &+ \sigma^2 e^{\lambda_0 - \nu_0} - \frac{\nu_0'' - \lambda_0''}{4} - \frac{(\nu_0' - \lambda_0')^2}{16} \\ &+ \frac{2w_0'^2}{r}(\frac{2}{r} - \lambda_0' + \nu_0') + \frac{8w_0'}{r^3}(w_0 - w_0^3)e^{\lambda_0} + \frac{(1 - 3w_0^2)}{r^2} e^{\lambda_0}\Bigg]\phi(r) = 0.\end{aligned} \tag{3.14}$$

IV. LATE-TIME TAILS IN THE SCHWARZSCHILD BACKGROUND

In the Einstein-Yang-Mills system, there exist two static, spherically symmetric black hole solutions: Schwarzschild and Reissner-Nordström solutions. Besides, there is the so-called colored black hole solution. But, the latter is dynamically unstable [14]. So it must decay to the Schwarzschild solution. Therefore as the final fates of the gravitational collapse of the YM field, the Schwarzschild and Reissner-Nordström black holes are two possibilities. In this section we discuss the case in which the final fate of the collapse is the Schwarzschild black hole.

In this case, we have

$$e^{\nu_0} = e^{-\lambda_0} = 1 - \frac{2m}{r}, \quad w_0 = \pm 1, \tag{4.1}$$

where m is the mass of the hole. Because the late-time behavior of perturbations is determined by the backscattering from the asymptotically far region, the late-time behavior is

dominated by the low-frequency contribution to the Green's function. Thus, as long as the observer is situated far from the black hole and the initial data has a considerable support only far from the black hole, the so-called asymptotic approximation is valid [5]. That is, a large-r (or equivalently, a low-σ) approximation is sufficient to study the asymptotic late-time behavior of the perturbations. Thus expanding (3.14), up to the terms $O(\sigma^2/r)$ and $O(1/r^2)$, yields

$$\left[\frac{d^2}{dr^2} + \sigma^2 + \frac{4m\sigma^2}{r} - \frac{2}{r^2}\right]\phi(r) = 0. \tag{4.2}$$

Introducing $\phi(r) = r^2 e^{i\sigma r}\tilde{\phi}(z)$ with $z = -2i\sigma r$, one may find the equation satisfied by $\tilde{\phi}$

$$\left[z\frac{d^2}{dz^2} + (4-z)\frac{d}{dz} - (2-2im\sigma)\right]\tilde{\phi}(z) = 0. \tag{4.3}$$

This is a confluent hypergeometric equation. It has two linearly independent solutions satisfying the requirement to construct the Green's function $\tilde{G}(y,x,\sigma)$ in (2.12). The two solutions are (for asymptotically far region $r >> m$)

$$\phi_1(r,\sigma) = Ar^2 e^{i\sigma r} M(2-2im\sigma, 4, -2i\sigma r), \tag{4.4}$$

and

$$\phi_2(r,\sigma) = Br^2 e^{i\sigma r} U(2-2im\sigma, 4, -2i\sigma r). \tag{4.5}$$

Here A and B are two normalization constants, $M(a,b,z)$ and $U(a,b.z)$ are two linearly independent solutions to the confluent hypergeometric equation (4.3).

Following [5,7], for simplicity, here we also assume that the initial data has a considerable support only inside the observer. Thus the branch cut contribution to the Green's function is

$$G(y,x;t) = \frac{1}{2\pi}\int_0^{-i\infty} \phi_1(x,\sigma)\left[\frac{\phi_2(y,\sigma e^{2\pi i})}{W(\sigma e^{2\pi i})} - \frac{\phi_2(y,\sigma)}{W(\sigma)}\right] e^{-i\sigma t} d\sigma. \tag{4.6}$$

Because $M(a,b,z)$ is a single-valued function, one has

$$\phi_1(r,\sigma e^{2\pi i}) = \phi_1(r,\sigma). \tag{4.7}$$

$U(a,b,z)$ is many-valued function including a branch cut. Using the formula

$$U(a,n+1,ze^{2\pi i}) = U(a,n+1,z) + 2\pi i\frac{(-1)^{n+1}}{n!\Gamma(a-n)}M(a,n+1,z), \tag{4.8}$$

where n is an integer, one may find

$$\phi_2(r,\sigma e^{2\pi i}) = \phi_2(r,\sigma) + \frac{i\pi B}{3A\Gamma(-1-2im\sigma)}\phi_1(r,\sigma). \tag{4.9}$$

Substituting (4.7) and (4.9) into (2.13), we have

$$W(\sigma e^{2\pi i}) = W(\sigma). \tag{4.10}$$

Using the fact that $W(\sigma)$ is independent of y, one can use the large-r limit of $\phi_i(r,\sigma)$ and reach

$$W(\sigma) = \frac{3iAB\sigma^{-3}}{4\Gamma(2-2im\sigma)}, \tag{4.11}$$

and

$$\frac{\phi_2(y,\sigma e^{2\pi i})}{W(\sigma e^{2\pi i})} - \frac{\phi_2(y,\sigma)}{W(\sigma)} = \frac{i\pi B}{3A\Gamma(-1-2im\sigma)}\frac{\phi_1(y,\sigma)}{W(\sigma)}. \tag{4.12}$$

Substituting them into (4.6), we get

$$\begin{aligned} G(y,x;t) &= \frac{2}{9A^2}\int_0^{-i\infty}\frac{\Gamma(2-2im\sigma)}{\Gamma(-1-2im\sigma)}\sigma^3\phi_1(x,\sigma)\phi_1(y,\sigma)e^{-i\sigma t}d\sigma \\ &\approx \frac{4im}{9A^2}\int_0^{-i\infty}\sigma^4\phi_1(y,\sigma)\phi_1(x,\sigma)e^{-i\sigma t}d\sigma. \end{aligned} \tag{4.13}$$

(1). *Late-time tail at future timelike infinity.* At future timelike infinity i^+ (where x, y $<< t$), we can use the $|\sigma|r << 1$ limit of the solution $\phi_1(r,\sigma)$. According to Eq. (13.5.5) of [15], one has

$$\phi_1(r,\sigma) \approx Ar^2. \tag{4.14}$$

Putting it into (4.13), we obtain

$$G(y,x;t) = \frac{32\pi m}{3}(xy)^2 t^{-5}. \tag{4.15}$$

(2). *Late-time tail at future null infinity.* At future null infinity $\mathscr{I}^+$, that is, near the region $y - x << t << 2y - x$, one may use the limit $|\sigma|x << 1$ limit of $\phi_1(x,\sigma)$ and the $|\sigma|y >> 1$ ($\mathrm{Im}\sigma < 0$) limit of $\phi_1(y,\sigma)$. Thus one has

$$\phi_1(x,\sigma) \approx Ax^2, \tag{4.16}$$

and

$$\phi_1(y,\sigma) \approx \frac{3!Ae^{i\sigma y+2im\sigma\ln y}}{\Gamma(2+2im\sigma)}e^{-i\pi(2-2im\sigma)}(-2i\sigma)^{-2+2im\sigma} + \frac{3!Ae^{-i\sigma y-2im\sigma\ln y}}{\Gamma(2-2im\sigma)}(-2i\sigma)^{-2-2im\sigma}, \tag{4.17}$$

by using Eq. (13.5.1) of [15]. Substituting them into (4.13), we have

$$G(y,x;t) = \frac{4m}{3}x^2(t-y)^{-3} \approx \frac{4m}{3}x^2u^{-3}. \tag{4.18}$$

(3). *Late-time tail along the black hole horizon.* Near the black hole horizon H^+, (4.4) does not satisfy the equation of the YM field (3.14). Considering (3.13) and (3.14), we have a suitable solution

$$\phi_1(y,\sigma) \approx Ce^{-i\sigma[y+2m\ln(y-2m)]}, \tag{4.19}$$

where C may depend on σ. But to match this solution to the solution for $y >> m$, C can be taken to be independent of σ [7]. Using (4.16) acts as $\phi_1(x,\sigma)$, we get

$$\begin{aligned} G(y,x;t) &= \Gamma_0 \frac{32m}{3} x^2 [t+y+2m\ln(y-2m)]^{-5} \\ &= \Gamma_0 \frac{32m}{3} x^2 v^{-5}, \end{aligned} \tag{4.20}$$

where Γ_0 is a constant.

Now some remarks are in order. First, we note that the equation (4.2) is same as the corresponding one for the scalar field with multipole moment $l=1$. Hence, these late-time behaviors (4.15), (4.18) and (4.20) of the YM field are same as those of massless neutral scalar field with $l=1$. For the latter see [5,7]. However, here we should point out that there exist some differences between them. For the scalar field in the Schwarzschild background, there is a centrifugal barrier term $l(l+1)/r^2$ in the effective potential. We are now considering the spherically symmetric excitation of the YM field, which corresponds to the s-wave of perturbations. The term $2/r^2$, corresponding to the $l(l+1)/r^2$ term for the scalar field, in (4.2) comes from the self-interacting term of the YM field, which can be seen clearly from (3.14). In this sense, the YM field therefore falls off faster the neutral scalar field. Second, when the background is the particle-like solution or colored black hole, asymptotically one has

$$e^{\nu_0} \approx e^{-\lambda_0} = 1 - \frac{2m}{r} + O(1/r^2), \quad w_0 = \pm 1 + O(1/r). \tag{4.21}$$

In this case, substituting them into (3.14) yields a same equation as (4.2). Therefore in the background of the particle-like solution or the colored black hole, the late-time behavior of the YM field is the same as that in the Schwarzschild background. This is expected because the late-time behavior of perturbations is determined by the nature of far region of backgrounds [2].

V. LATE-TIME TAILS IN THE REISSNER-NORDSTRÖM BACKGROUND

In this section we discuss the case when the background is the Reissner-Nordström black hole. In the Einstein-Yang-Mills system, it has been shown that the charged, spherically symmetric black hole solution must be the Reissner-Nordström solution and the regular monopole and dyon do not exist. The no-hair theorem therefore holds for the charged black hole [16]. Thus, in this case we have

$$e^{\nu_0} = e^{-\lambda_0} = 1 - \frac{2m}{r} + \frac{g^2}{r^4}, \quad w_0 = 0, \tag{5.1}$$

where $g^2 = 1$ is the magnetic charge of the solution. Expanding (3.14), in the far region, reduces to

$$\left[\frac{d^2}{dr^2} + \sigma^2 + \frac{4m\sigma^2}{r} + \frac{1}{r^2}\right]\phi(r) = 0. \tag{5.2}$$

Introducing

$$\phi(r) = r^{\frac{1}{2}} e^{i(\frac{\sqrt{3}}{2}\ln r + \sigma r)} \tilde{\phi}(z), \quad z = -2i\sigma r, \tag{5.3}$$

from (5.2) we obtain

$$\left[z\frac{d^2}{dz^2} + (1 + i\sqrt{3} - z)\frac{d}{dz} - \left(\frac{1}{2} - \frac{i}{2}(4m\sigma - \sqrt{3}) \right) \right] \tilde{\phi}(z) = 0. \tag{5.4}$$

Once again, this is a confluent hypergeometric equation. Thus we have two equations satisfying the requirement to construct the Green's function

$$\phi_1(r,\sigma) = A r^{\frac{1}{2}} e^{i(\frac{\sqrt{3}}{2}\ln r + \sigma r)} M[\frac{1}{2} - \frac{i}{2}(4m\sigma - \sqrt{3}), 1 + i\sqrt{3}, -2i\sigma r], \tag{5.5}$$

$$\phi_2(r,\sigma) = B r^{\frac{1}{2}} e^{i(\frac{\sqrt{3}}{2}\ln r + \sigma r)} U[\frac{1}{2} - \frac{i}{2}(4m\sigma - \sqrt{3}), 1 + i\sqrt{3}, -2i\sigma r]. \tag{5.6}$$

For these two solutions, we have

$$\phi_1(r, \sigma e^{2\pi i}) = \phi_1(r,\sigma), \tag{5.7}$$

$$\phi_2(r, \sigma e^{2\pi i}) = e^{2\pi\sqrt{3}} \phi_2(r,\sigma) + \frac{A}{B} \frac{(1 - e^{2\pi\sqrt{3}})\Gamma(-i\sqrt{3})}{\Gamma[\frac{1}{2} - \frac{i}{2}(4m\sigma + \sqrt{3})]} \phi_1(r,\sigma). \tag{5.8}$$

The Wronskian satisfies

$$W(\sigma e^{2\pi i}) = e^{2\pi\sqrt{3}} W(\sigma). \tag{5.9}$$

Using the asymptotic behaviors of $M(a,b,z)$ and $U(a,b,z)$ as $|z| \to \infty$, we get

$$W(\sigma) = -AB e^{-\frac{\sqrt{3}}{2}\pi - i\sqrt{3}\ln 2} \sigma^{-i\sqrt{3}} \frac{\Gamma(1 + i\sqrt{3})}{\Gamma[\frac{1}{2} - \frac{i}{2}(4m\sigma - \sqrt{3})]}. \tag{5.10}$$

Further we obtain

$$\frac{\phi_2(y, \sigma e^{2\pi i})}{W(\sigma e^{2\pi i})} - \frac{\phi_2(y,\sigma)}{W(\sigma)} = \frac{B}{A} \frac{\Gamma(-i\sqrt{3})}{\Gamma[\frac{1}{2} - \frac{i}{2}(4m\sigma + \sqrt{3})]} \frac{(e^{-2\pi\sqrt{3}} - 1)}{W(\sigma)} \phi_1(y,\sigma). \tag{5.11}$$

Putting it into (4.6) we reach

$$\begin{aligned} G(y,x;t) &= \frac{1}{2\pi A^2} \frac{(1 - e^{-2\pi\sqrt{3}})}{e^{-\frac{\sqrt{3}}{2}\pi - i\sqrt{3}\ln 2}} \frac{\Gamma(-i\sqrt{3})}{\Gamma(1 + i\sqrt{3})} \\ &\times \int_0^{-i\infty} \frac{\Gamma[\frac{1}{2} - \frac{i}{2}(4m\sigma - \sqrt{3})]}{\Gamma[\frac{1}{2} - \frac{i}{2}(4m\sigma + \sqrt{3})]} \sigma^{i\sqrt{3}} \phi_1(x,\sigma)\phi_1(y,\sigma) e^{-i\sigma t} d\sigma \\ &\approx \frac{1}{2\pi A^2} \frac{(1 - e^{-2\pi\sqrt{3}})}{e^{-\frac{\sqrt{3}}{2}\pi - i\sqrt{3}\ln 2}} \frac{\Gamma(-i\sqrt{3})}{\Gamma(1 + i\sqrt{3})} \frac{\Gamma(\frac{1}{2} + i\frac{\sqrt{3}}{2})}{\Gamma(\frac{1}{2} - i\frac{\sqrt{3}}{2})} \\ &\times \int_0^{-i\infty} \sigma^{i\sqrt{3}} \phi_1(x,\sigma)\phi_1(y,\sigma) e^{-i\sigma t} d\sigma. \end{aligned} \tag{5.12}$$

(1). *Late-time tail at future timelike infinity.* At the future timelike infinity i^+, as in the Schwarzschild background, we can use the limit $|\sigma|x << 1$ and $|\sigma|y << 1$ for $\phi_(x, \sigma)$ and $\phi_2(y, \sigma)$. That is, we can take

$$\phi_1(r, \sigma) \approx A r^{\frac{1}{2}} e^{i\frac{\sqrt{3}}{2}\ln r}. \tag{5.13}$$

Substituting it into (5.12), we find

$$G(y, x; t) = -i\frac{(1 - e^{-2\pi\sqrt{3}})\Gamma(-i\sqrt{3})}{2\pi e^{-\pi\sqrt{3} - i\sqrt{3}\ln 2}} \frac{\Gamma(\frac{1}{2} + i\frac{\sqrt{3}}{2})}{\Gamma(\frac{1}{2} - i\frac{\sqrt{3}}{2})}(xy)^{\frac{1}{2}} e^{i\frac{\sqrt{3}}{2}\ln xy} t^{-1-i\sqrt{3}}. \tag{5.14}$$

(2). *Late-time tail at future null infinity.* At the future null infinity $\mathscr{J}^+$, we can use the $|\sigma|x << 1$ limit for $\phi_1(x, \sigma)$, while the $|\sigma|y >> 1$ limit for $\phi_1(y, \sigma)$. That is, we take (5.13) for $\phi_1(x, \sigma)$, and

$$\begin{aligned} \phi_1(y, \sigma) = A y^{\frac{1}{2}} e^{i(\frac{\sqrt{3}}{2}\ln y + \sigma y)} & \left\{ \frac{\Gamma(1 + i\sqrt{3}) e^{-i\pi[\frac{1}{2} - \frac{i}{2}(4m\sigma - \sqrt{3})]}}{\Gamma[\frac{1}{2} + \frac{i}{2}(4m\sigma + \sqrt{3})]} (-2i\sigma y)^{-\frac{1}{2} + \frac{i}{2}(4m\sigma - \sqrt{3})} \right. \\ & \left. + \frac{\Gamma(1 + i\sqrt{3}) e^{-2i\sigma y}}{\Gamma[\frac{1}{2} - \frac{i}{2}(4m\sigma - \sqrt{3})]} (-2i\sigma y)^{-\frac{1}{2} - \frac{i}{2}(4m\sigma + \sqrt{3})} \right\}, \end{aligned} \tag{5.15}$$

using Eq. (13.5.1) of [15]. Substituting them into (5.12), in this case we obtain

$$G(y, x; t) = -i\frac{(e^{\pi\sqrt{3}} - e^{-\pi\sqrt{3}})}{2\pi\sqrt{2} e^{-i\frac{\sqrt{3}}{2}\ln 2}} \frac{\Gamma(-i\sqrt{3})\Gamma(1 + i\frac{\sqrt{3}}{2})\Gamma(\frac{1}{2} + i\frac{\sqrt{3}}{2})}{\Gamma(\frac{1}{2} - i\frac{\sqrt{3}}{2})\Gamma(\frac{1}{2} + i\frac{\sqrt{3}}{2})} x^{\frac{1}{2}} e^{i\frac{\sqrt{3}}{2}\ln x} u^{-\frac{1}{2} - i\frac{\sqrt{3}}{2}}. \tag{5.16}$$

(3). *Late-time tail along the black hole horizon.* Near the black hole horizon H^+, once again, the solution (5.5) does not satisfy the equation (3.14). The appropriate solution should be

$$\phi_1(y, \sigma) \approx C e^{-i\sigma\left[y + \frac{1}{2\kappa}\ln(y - r_+)\right]}, \tag{5.17}$$

where r_+ is the horizon radius and κ is the surface gravity on the black hole horizon. Taking (5.13) as $\phi_1(x, \sigma)$, we finally obtain

$$G(y, x; t) = -i\Gamma_0 \frac{(1 - e^{-2\pi\sqrt{3}})\Gamma(-i\sqrt{3})}{2\pi e^{-\pi\sqrt{3} - i\sqrt{3}\ln 2}} \frac{\Gamma(\frac{1}{2} + i\frac{\sqrt{3}}{2})}{\Gamma(\frac{1}{2} - i\frac{\sqrt{3}}{2})} x^{\frac{1}{2}} e^{i\frac{\sqrt{3}}{2}\ln x} v^{-1-i\sqrt{3}}. \tag{5.18}$$

where Γ_0 is a constant.

Comparing the late-time tails (5.14), (5.16) and (5.18) in the Reissner-Nordström black hole background with those (4.15), (4.18) and (4.20) in the Schwarzschild black hole background, we can find easily that there are a lot of differences between them. First, we notice that the damping exponents are different. The damping exponent in the Schwarzschild background is always larger than the corresponding one in the Reissner-Nordström background. In this sense, the YM hair decays in the Schwarzschild background faster than in the Reissner-Nordström background. Another important difference is the occurrence of an oscillatory factor in the Reissner-Nordström background. This oscillatory factors are all present

for the three late-time tails. Note that for charged scalar field, the oscillatory factor occurs only for the late-time tail along the black hole horizon. Second, the late-time tails of the YM field are also of qualitative differences. It can be observed that the late-time tails (4.15), (4.18), and (4.20) are all proportional to the mass of the black hole. This implies that the late-time behavior of the YM field is an effect of spacetime curvature in the Schwarzschild background. However, the late-time tails in the Reissner-Nordström background have nothing to do with the mass or charge of the hole. Actually, the late-time behavior of YM field in the Reissner-Nordström background is dominated by the self-interaction of the YM field, an effect in a flat spacetime. The causes responsible for the different results are clear. From (3.14) it can be seen that the self-interacting term of YM field provides the excitation with a barrier ($2/r^2$) in the Schwarzschild case, but with a well ($-1/r^2$) in the Reissner-Nordström background. In fact, it is the difference that makes the existence of the particle-like solution and colored black hole and nonexistence of the regular monopole and dyon in the Einstein-Yang-Mills theory with the su(2) gauge group.

VI. CONCLUSIONS

We have investigated the late-time evolution of the Yang-Mills field in its own gravitational backgrounds: Schwarzschild and Reissner-Nordström solutions. The Green's functions describing the late-time tails are calculated at three asymptotic regions: the future timelike infinity i^+, the future null infinity $\mathscr{J}^+$ and the outer horizon H^+ of black holes. The late-time evolution is different in the two backgrounds, quantitatively and qualitatively. When the background is the Schwarzschild solution, the late-time tails of the YM field are the same as those of neutral massless scalar field with multipole moment $l = 1$. Note that the perturbations considered in this paper are spherically symmetric excitations. In this sense, the YM hairs die off faster than scalar hairs. Note that all late-time tails are proportional to the mass of the black hole. Therefore the late-time evolution of YM field in the Schwarzschild background is dominated by the spacetime curvature. This is the same as the neutral massless scalar field. However, there still exists a essential difference. For the case of scalar field, the centrifugal barrier term $l(l+1)/r^2$ in the effective potential is an effect of angular momentum. In our case, the term $2/r^2$, which corresponds to the term $l(l+1)/r^2$ of scalar field, in (4.2) comes from the self-interacting term of the YM field.

When the background is the Reissner-Nordström solution, some interesting changes occur. The late-time tails (5.14), (5.16) and (5.18) have not only smaller damping exponents, compared to those in the Schwarzschild background, but also an oscillatory factor. This implies that the YM hair falls off in the Schwarzschild background faster than in the Reissner-Nordström background. This oscillatory factor is present for all three asymptotic region with different periods. The period is $2\pi/\sqrt{3}$ for the late-time tail at the future timelike infinity and along the black hole horizon, and $4\pi/\sqrt{3}$ for the tail at the future null infinity. These late-time behaviors are dominated by the self-interacting term of the YM field. They are an effect in a flat spacetime. The cause resulting in the different results is that the self-interacting term of YM field gives in the effective potential an attractive term for the Reissner-Nordström background, while a repulsive term for the Schwarzschild background.

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